designing trap antennas: a new approach

Symmetrical design increases gain and directivity

Most Amateurs don't realize that the original design of a trap antenna^{$1,2,3,4$} represents just one particular case, and is not the best possible choice of the many designs available. According to the references, a dipole designed to resonate at two frequencies, fl and f2, contains parallel RLC circuits $-$ or traps $$ that are resonant at the upper frequency, f2, and physically separated by a half wavelength (at f2). This approach I call the "classic" design. At f2 these circuits offer a very high impedance and, in effect, disconnect the outer portions of the antenna. The inner portion acts as a half-wave dipole. With the total antenna length made equal to one half wavelength at the lower design frequency, fl, the antenna would resonate at this frequency if the traps presented a short circuit. In practice, however, the traps introduce an inductive reactance at all frequencies lower than f2. To compensate, the total length of the antenna is slightly shortened in order to be resonant at fl.

Treatments of this subject in reference books usually ignore other existing designs and imply incorrectly that this description applies to all trapped antennas.

Extending this design approach to three bands (10, 15, and 20 meters) would require the two sets of traps to resonate at 21 and 28 MHz and be separated by approximately 3 feet (on each half-dipole element). Some of the first commercially manufactured triband antennas actually did use separated traps, but those generally available today have such pairs of traps built into a single assembly.

A vertical trap antenna, just slightly longer than a quarter wavelength at the upper frequency f2, was discussed in an earlier article; the principle upon which that antenna was based can be applied to dipoles as well.⁵ In this case, the trap is placed at the base of the antenna and its parameters are selected to produce the correct inductive reactance for resonance at the lower frequency fl. At f2 the antenna is slightly longer than a quarter wavelength, and the trap provides the necessary *capacitive* reactance for resonance to be achieved.

To lower the resonant frequency of a fixed-length dipole, inductance must be added. Conversely, capacitance must be added to raise its frequency. A parallel RLC circuit can provide the required reactances at fl and f2. (A series RLC circuit, which has **4** capacitive reactance at low frequencies and inductive reactance at high frequencies, cannot be used in this application.

There's wide variation in the positioning of the traps, which affects the properties of the antenna (directivity, bandwidth, efficiency, and trap resonant frequencies). Mathematically speaking, the design of a trapped antenna is over-determined since there are four parameters to be adjusted in order to meet the two resonance conditions at f1 and f2: total length: trap position; trap resonant frequency, fO (when removed from the antenna) and the value of the capacitance, C.

Without further information, it's impossible to say how these parameters are to be selected in order to obtain the best performance. The classic design isn't optimum because the outer portions of the dipole have little current at the upper frequency, f2. A design that causes a significant current to flow in these outer portions can improve directivity.

In addition, minimum loss reactances should be used. The capacitive loading required by a single-band antenna longer than a half wavelength can usually be supplied with negligible loss. However, inductive loading required for a single-band short dipole usually adds

By Yardley Beers, WBJF, 740 Willowbrook Road, Boulder, Colorado 80302

some losses. With a two-band trap antenna, the losses are usually greater at both frequencies than if singleband loading is used. Because losses tend to be greater if the trap resonance is near one of the operating frequencies, I'll discuss a design in which **fO** is as far removed from the operating frequencies, fl and f2, as possible, and in which the traps have equal impedances at these two frequencies. For this to occur, the trap resonant frequency, **fO,** must be equal to the geometrical average of fl and f2 - that is, equal to the square root of their product. The reactances at the two design frequencies are equal in magnitude but different in sign, and the resistances are equal. I call this approach the "symmetrical" design.

a dual-band antenna

To illustrate this principle, two separate two-band trap dipoles were built; the first for 18.1 and 24.9 MHz, and the second for the 14- and 21-MHz bands. Actually, I have built two of the former; the first had minimum SWRs of 1.5 in the 18.1-MHz band and 1.4 in the 24.9-MHz band. After I had dismantled it, I improved my computer programs for designing these antennas and built the 14/21-MHz antenna, with which I obtained minimum SWRs of 1.1 on both bands. I reworked the calculations for the $18.1/24.9$ -MHz antenna and got slightly different values. I decided that although the original antenna was still useful, I hadn't trimmed it to best advantage. 1 built the second antenna with significantly different dimensions and, as shown in **fig. 1,** it has minimum SWR of 1.1 on both bands.

18.1/24.9-MHz dual-band antenna

Both antennas were developed for design frequencies of 18.118 and 24.94 MHz with a trap design frequency of 21.257 MHz, using nominal trap capacitances of 50 pF. After the second version of the dual-bander was built, the trap capacitances were measured at 51.6 pF. The trap resonances measured, on the average, 21.27 MHz. The inductors consisted of 13 turns of No. 18 plastic-coated wire wound on 112-inch diameter plastic rod. Since I operate at low power (about 100 watts) I was able to use No. 18 wire, but I advise those readers who intend to operate at higher power to build inductors with heavier wire. I didn't measure the Qs of the coils, but I assume that 200 is a reasonable value. (Resonance is independent of the **Q.)** Using these values and **eqns. 8,** 10, and 11 (see **appendix** I), I found the magnitude of the reactances and the resistances to be 449 and 7.0 ohms, respectively.

The total antenna length is 20.0 feet. The traps are 13.8 feet apart, symmetrically placed about the center feedpoint. The earlier antenna, with similar traps, had dimensions of 20.7 feet (length) and 15.3 feet (trap

separation). For comparison, the (unloaded) lengths of half-wave dipoles for these design frequencies, as given by **eqn. 6** (see **appendix** I), are, respectively, 25.9 feet and 18.8 feet. The wire is No. 14 gauge. Standing wave measurements relative to a 50-ohm transmission line, with the antenna 20 feet above ground, are shown in **fig. 1.** The measurements take line losses into account.

Before building the second antenna, I followed the mathematical procedure discussed in **appendix 2,** which gave the following results: for S1 (the distance from the center feedpoint to one trap), 6.90 feet; for S2 (the distance from a trap to the end), 3.88 feet. With these dimensions, the SWR pattern at 24.94 **MHz** wasn't very different from that shown in **fig. 1,** but the fl SWR minimum of 1.2 was at 17.4 MHz, about 5 percent too low in frequency. I tried trimming both S1 and S2. Finally I retained the original value of S1. but I reduced S2 by 0.8 feet (9.5 inches) for a final value of 3.08 feet. These dimensions apply to **fig. 1.**

Moxon gives a method for estimating the efficiency of trapped antennas.⁶ It's necessary to measure or assume values for the radiation resistance. The minimum SWR at both frequencies is 1.1, which implies that the total resistance at the drivepoint is between the limits $50/1.1$ = 45.5 ohms and 50×1.1 = 55 ohms. Because the radiation resistance is larger for dipoles greater than one half wavelength and less for those less than one half wavelength, I assumed the lower (resistance) value for fl and the upper one for f2.

The power radiated is the current at the center multiplied by the square of the current at the center. The power lost in the traps is their resistance multiplied by the square of the current at the traps, which is smaller than at the center. The total resistances of the traps is actually 14 ohms, but a calculation shows that the losses are as though the trap resistances were at the center and had the values of 5.8 and 2.0 ohms, respectively, at f_1 and f_2 . After subtracting off these transformed loss resistances, I obtain estimates for the radiation resistances at the two frequencies as 45 $- 5.8 = 39.2$ and $55 - 2.0 = 53$ ohms, respectively. The efficiencies are then respectively $39.2/45 = 87\%$ and $53/55 = 96\%$.

However, since the radiation resistance of a halfwave dipole is about 72 ohms, one would expect the radiation resistances to be slightly higher than the estimates given above; consequently, working with the same trap resistances, efficiency should actually be higher.

15120-meter dual-band antenna

The design frequencies for this antenna are, respectively, $f1 = 14.15$, $f2 = 21.2$, and $f0 = 17.32$ MHz. The trap capacitors have measured values of **52** pF, on the average. The inductors are seven and a half

NEW COMPUTER BOOKS

YOUR COMMODORE 64: A GUIDE TO THE C-64 COMPUTER YOUR COMMODORE 128: A GUIDE TO THE

C-128 COMPUTER

These books cover in great detail the best selling Com-
modore C-64 and C-128 home computers. You get a
complete introduction to the operating sytems used, BAS-IC tutorials, graphics, sound and much more. Also discussed are hardware and peripheral considerations. The -128 book covers C-64 emulation, extended memory, CP/M mode, mouse, ram disk, printers and modems. Excellent source books for beginners and experts alike. *O* 1985 1st Edition

I **10s-C Save \$4.95 \$24.95**

MS-DOS USER'S GUIDE

by Chris DeVoney Most an essential addition to their computer library. Includes a full explanation of MS-DOS commands in clear, concise language and examples of command syntax. Hints on command usage and explanations of the hierarchial directory and 110 redirection will enable you to get maxi- mum benefit from your computer investment. For novice and expert users. © 1987 2nd Edition 330 pages.
 QUE-061 Softbound \$21 **LIPUE-061 Softbound \$21.95**

PC SECRETS: TIPS FOR POWER PERFORMANCE

by James Kelly

Here's one of those unheralded gems we stumbled upon recently. This nifty book is jam-packed with ideas and
suggestions on how to get more out of your PC-DOS or
MS-DOS computer. Not a tutorial; more for the intermediate user who is looking to get more speed and efficiency. Improve your keyboard, enhance your display,
organize your files, and manage your printer better than
ever before. You'll be amazed at what this book can add
to your PC. Also covers Lotus 1-2-3 and Wordstar. ©
1985 1st

¹**l0S.PCS Softbound \$16.95**

APPLE II USER'S GUIDE, Apple Plus and II series

by Poole, McNuff and Cook
All time Apple II best seller! Now available in updated third edition. Learn from the experts how to get the most from your Apple home computer and peripherals. You also get a complete explanation on how to use DOS 3.3 and Pro-DOS. Easy-to-use tutorial explanation of BASIC programming will teach you how to use all the sound and graphics capabilities as well as the Apple Ile high resolution graphics. This book is worth it's weight in gold!
 CO 1985 1st Edition 512 pages
 Softbound \$18.95
 Softbound \$18.95

 $$

We're really proud of the next two books! Doug
was Jim Fisk's right hand man during the early
seventies. His first computer book, The Introduc-
tion to TURBO Pascal, quickly went best seller.
The TURBO Library is an invalu **TURBO user's libraries.**

INTRODUCTION TO TURBO PASCAL by Doug Stlvison WAlKWJ (ex Ham Radio assistant editor)

Thousands have learned Pascal programming with this
popular best seller. As a tutorial this book enhances the
unique aspects of Turbo Pascal by concentrating on the extended applications capabilities offered. Includes graph-
ics, look-up tables, word processor to typesetting equip-
ment conversion tables, TTS to ASCII conversion and fast sorVsearch routines. O 1985 1st Edition 268 pages [**ISY-269 Solbound \$14.95**

TURBO PASCAL LIBRARY by Doug Stivlson WAlKWJ

Perfect compliment to the Turbo Pascal Introduction book listed above. Stivison shares his extensive collection of proven programs and will save experienced programmers
time and illustrate to beginners good programming techincourse and Turbo versions of standard algorithims. Includes games, systems utilities, and routines for business
and engineering applications. © 1986 1st Edition 350

pages
[]**SY-330 Softbound \$14.95**
Please Enclose \$350 to cover **Greenville, NH 03048**

fig. 1. SWR vs. frequency for trapped dipole for use in 18.068-18.168 MHz and 24.890 -24.990 MHz bands. The antenna is 20 feet, 10 inches long and has traps placed 15 feet, **2 inches apsrt, equidistant from the center feedpoint. The traps use 50-pF capacitors and are self-resonant at 21.2 MHz.**

FERROXCUBE DEVICE

121 Brown Street * Dayton, Ohio 45402 * (513) 220-9677

POWER SPLITTER/COMBINER
2-30 MHz, 600 Waits (1 Part of 4 Port)

lConcepts Inc.

Communication

 $rac{1}{20}$

.8 49.95 KH
.8 59.95 KH

KEMET CHIP CAPACITORS
METALCLAD MICA CAPACITORS
SEMICONDUCTORS
RF POWER TRANSISTORS

 $~136$

For detailed information, please
call or write for our free catalog

ATV-3 420-450 MHz GaAs-Fet
ATV-4 902-928 MHz GaAs-Fet

Add \$ 2.00 For Shipping and Handling

n. Kit ar Assembled/Tested

VISA

turns of No. 18 plastic-coated wire on 1-inch plastic pill bottles. The wire spacing was varied to adjust the resonant frequencies of the traps. During coil construction, the half turn was left slack for fine tuning. The average measured resonant frequency was 17.41 MHz. The wire used in the antenna is No. 12 gauge.

Appendix 2 provides a detailed discussion of calculations which suggested the initial dimensions S1 $= 7.69$ feet and S2 $= 5.41$ feet. After I trimmed them experimentally to $S1 = 7.36$ feet and $S2 = 5.24$ feet, I obtained the standing wave pattern shown in **fig.** 2.

As the length of a loaded dipole, measured in wavelengths, increases, the bandwidth increases. Although a wider bandwidth is expected at f2 than at fl, I can't explan why bandwidths I've observed at f2 are very much wider than those at fl. These different bandwidths showed considerable sensitivity to changes in length while trimming; in each of the three cases I've dealt with, the standing wave pattern at f2 has changed hardly at all when I changed the dimensions, but a change of only a few inches made a significant change in the fl resonance frequency. It's likely that the fl resonance is very susceptible to changes in antenna location.

theory of the symmetrical design

The selection of a trap antenna's dimensions can be made almost entirely by experiment. The only mathematics one needs to build a two-band antenna using the symmetrical design is to calculate fO by taking the square root of the product of fl and f2. After installing the traps and adjusting the antenna length and trap location, a working antenna is produced. The mathematical details can usually save the designer some time by providing at least a "starting point."

In designing a trap antenna, the following conditions must be satisfied: it must resonate at fl; it must resonate at f2; the trap resonant frequency must be the geometric average of fl and f2. The trap resistances at fl and f2 are equal, and the reactances are equal in magnitude but opposite in sign. Arbitrarily, I chose 50 pF as a convenient value for the trap capacitances. (The effect of using other values is also considered.) The required vdlue of the trap inductances, L, can then be determined by **eqn.** 7.

Most of the necessary formulas are listed in **appendix** 1, with detailed examples provided in **appendix** 2. The condition for resonance is that the total reactance as referred to any point on the antenna is zero. It is axiomatic that **if** the total reactance at **any** one point is zero, then it is zero at all other points. For convenience, I chose the trap location as a reference point and considered only half the dipole at a time.

The antenna and the ground can be thought of as forming a transmission line. Although the center is connected to the feedline, I assume that its input impedance, as viewed at the location of the trap, is the same (positive) reactance, XI, as it would be if the center were connected to ground. Calculate this with **eqn. 15.** Similarly, I consider the part between the trap and the end as a section of open-circuited transmission. Its impedance is given by the (negative) reactance, X2, in **eqn.** 17.

If S1, the distance from the center to the trap, and if S2, the distance from the trap to the open end, are a quarter wavelength long, then $X1 + X2 = 0$, in**dependent** of **the value** of *SI.* This configuration is well known as a resonator.

For resonance using arbitrary lengths of **S1** and S2, it's necessary to introduce a reactance X such that:

$$
X + XI + X2 = 0 \tag{1}
$$

Various values of S1 and S2 are tried until the equation is satisfied at both fl and f2. (See **appendix** 2.)

As **a** simplification, a characteristic impedance of 575 ohms was calculated for a transmission line consisting of a horizontal 12-gauge wire parallel to and 20 feet above a perfectly conducting groundplane **(eqn.** 14). Though good for a transmission line, this formula neglects the effect of radiation, and, of course, radiation is the primary purpose of the antenna.

The calculation of characteristic impedances of radiating antennas is more complicated.'

parallel RLC circuit properties

A complete understanding of the design of trap antennas requires a knowledge of parallel RLC circuits. Graphs of impedance, series resistance, and series reactance are shown as a function of frequency f in **fig.** 3. These normalized curves require that quantities plotted on the vertical scale be multiplied by the value of *, and the frequency shift measured from the* resonant frequency be given in units of **D,** where

$$
D = f0/2Q \tag{2}
$$

Losses in the inductor and capacitor are represented by a resistance, R , where:

$$
R = \frac{500,000Q}{\pi Cf0} \text{ ohms} \tag{3}
$$

and C is the capacitance in picofarads, and f_0 is the resonant frequency in MHz.

To apply these ideas to the traps in the 18.1/24.9-MHz antenna, $f0 = 21.257$ MHz, $C = 51.6$ pF, R = 29,002 ohms, and D = 0.053 MHz **(53** kHz). The reactance has small positive values at low frequencies and increases to a maximum of $0.5 R = 14,501$ ohms at a normalized value of -1 (21.204 MHz), after which it decreases to zero at resonance. Then it decreases to a negative maximum -0.5 R = $-14,001$ ohms, at $+1$ (21.310 MHz) and approaches zero at very large frequencies.

The design frequencies, fl and f2, correspond respectively to -70.6 and $+58.4$, and lie very far off the graph in fig. 3. The values of the impedances, resistances, and reactances cannot be obtained from the graph and must be calculated by formulas such as those shown in appendix **1.** The reactances, as stated previously, are \pm 449 ohms and the resistances are 7.0 ohms. Consequently, the circuit can be considered as a low-loss reactance.

Since the reactance goes through zero at resonance, there are two other frequencies $-$ very close (about 10 k Hz) $-$ at which the desired reactance of $±$ 449 ohms can be obtained. There the resistance is very close to its maximum value. Such a reactance would hardly be considered low-loss! This is the situation encountered when such a circuit is used as a trap at f2 in the classical design. In such a situation the RLC circuit becomes a "trap" in another sense of the word: as an absorber of power.

The curves shown in fig. 3 assume that Q is at least 10 and preferably much larger, as is likely to be the case in practice. The errors resulting from this assumption are then negligible. This assumption simplifies the math. When **Q** is small, it is necessary to draw individual curves. Equations **8.9.10.** and **11** don't have this limitation.

Another assumption is that the losses in the circuit are represented by a resistor, R, whose value is given by eqn. **3,** and that its value is independent of frequency. Strictly speaking, this assumption is unrealistic. Most of the losses are in the inductor, and they depend upon frequency in a complicated way because of skin effect and distributed capacitance.

For physical reasons, air (dielectric) capacitors often aren't practical to use in antenna traps — but they are low-loss. The losses in solid dielectric capacitors are generally lower than those in the **inductors,** but they (the losses) mustn't be overlooked. While an air-core inductor has the shape to dissipate heat, a solid-state capacitor does not; a small amount of power dissipated in it causes its temperature to rise and its capacitance to change. If the effect is large, the antenna is detuned when the power level is raised. Thus, a solidstate capacitor has a current limitation as well as a voltage limitation. Usually one that has a high voltage rating can also pass a lot of current, but this isn't always true. Capacitors with thin pigtails, even with high voltage ratings, must be avoided. Sometimes it's necessary to use more than one capacitor in series or parallel to get sufficient current carrying capability.

conclusion

The main purpose of this article is to review the principles of design of trapped antennas and to show that there are alternative designs that are superior to the classic one. I've illustrated the principles by showing

 $\ddot{}$

the design of practical antennas for the 18.1- and **24.9-MHz** bands, and for the 14- and 21-MHz bands.

A central point of the discussion is that the operating and trap frequencies should not coincide. Equation **12** shows that with far-off resonance the series resistance decreases with the square of the difference between the operating and trap resonant frequencies. A further argument for keeping the trap resonant frequency remote from the operating frequency is the fact that the same RLC circuits used in the 18.1/24.9-MHz antenna could be used as traps in a 14/21-MHz dual-band antenna of classic design. The total trap resistance at the new f2 (21 MHz) is about 58,000 ohms. In considering such an antenna, Moxon⁶ estimates that the radiation resistance, when transformed to the trap location, is about 32,000 ohms. Therefore, the efficiency is about $(32/60) = 53$ percent, whereas the efficiency with the symmetrical design at $f2 = 24.94$ MHz is 96 percent. Furthermore, with the symmetrical design there is significant current in the outer portions, and the directivity is greater. Therefore, at the upper frequency, f2, there is a significant advantage of the symmetrical over the classic design. With the classic design, the lower frequency, fl, is further from resonance than with the symmetrical design, and therefore the loss resistance is lower and the efficiency is higher. With the symmetrical design, it is already very good $-$ 87 percent $-$ so this advantage is not important.

I haven't tried to optimize the value of the trap capacitances. Equation **13** shows that the resistances of the traps' far-off resonance are inversely proportional to the product of the capacitance and the **Q** of the traps. With the present values of 50 pF and a **Q** of 200, they're about the largest acceptable. If the capacitances were lowered to 25 pF, the resistances would be double and would cause a significant reduction in efficiency at both operating frequencies. If they were made 100 pF, there would be an insignificant improvement in the efficiency.

The principle of symmetrical design should be applicable to a 14/21 /28-MHz triband dipole. The traps should be resonant in the vicinity of 17 MHz and 24 MHz, respectively. Efficiency and directivity would be greater. However, there would be a problem in extending the design to many bands; the radiation resistance would be different on the lowest and highest frequency bands, and an elaborate impedance matching network would be required to get a good match to the transmission line for all bands.

This article has dealt only with a single dipole. This approach can be applied to multi-element antennas with both driven and parasitic elements.

acknowledgments

I wish to thank Les Moxon, GGXN, Francis M.

л

New rigs and old favorites, plus the best essential accessories for the amateur.

CALL FOR ORDERS 1 (800) 231-3057
1-713-520-7300 OR 1-713-520-0550
TEXAS ORDERS CALL COLLECT ALL ITEMS ARE GUARANTEED OR SALES **PRICE REFUNDED**

USED EQUIPMENT

All equipment, used, clean, with 90 day warranty and 30
day trial. Six months full trade against new equipment. Sale price refunded if not satisfied. Call for latest used gear list

 (800) 231-3057

AMPHENOL

O ICOM

TOWER ACCESSORIES

POLICIES

today

Call

products in stock

Belden

Bird and

Minimum order \$10.00. Mastercard, VISA, or C.O.D. All prices FOB Houston, except as noted. Prices subject to change without notice. Items subject to prior sale. Call anytime to check the status of your order Texas residents add sales tax. All items full factory warranty plus Madison warranty

DON'S CORNER

New! "World Radio Report" for SWL & Hams. Full of info.
\$2.50/month. Try 30 meters? N5JJ Houston has 101 countries. Have more? Contact Don at Madison -130

 -129

Dukat, KGNL, and Oswald G. Villard, Jr., WGQYT, for their valuable suggestions.

appendix 1

basic equations

The formulas have been used for calculations discussed in the text. The wavelength

$$
\lambda = 299.8/f \text{ in meters} \qquad (4)
$$

$$
= 983.6/f \text{ in feet, with the frequency } f \text{ in MHz.} \qquad (5)
$$

The length of a half-wave dipole is

$$
G = 468/f \text{ in feet.}
$$
 (6)

Equation 6 gives a length which is 95 percent of a half wavelength as given by eqn. 5 because of corrections for end effects. This simplification valid for hf wire antennas depends on the ratio of conductor diameter to wavelength. At VHF, where the ratio is usually smaller, the correction is larger.

The inductance L (in μ H) required to resonate with a capacitor C (in pF) at a frequency **fO** (in MHz) is

$$
L = \frac{l}{4\pi^2} \frac{l}{C (f_0)^2}
$$
 (7)

It is convenient to define a new quantity, **t**

$$
t = Q \cdot \frac{f^2 - f_0^2}{f \cdot f_0} \tag{8}
$$

where *f0* is the resonant frequency of the RLC circuit and *f* is any arbitrary frequency. The impedance is given by

$$
Z = \frac{R}{\sqrt{I + t^2}}
$$
 (9)

The series resistance is given by

$$
R_s = \frac{R}{l + l^2} \tag{10}
$$

and the reactance is given by

$$
X_s = \frac{R \ t}{1 + t^2} \tag{11}
$$

To understand the behavior of traps far from resonance, it is desirable to derive approximate expressions for the resistance and reactance.

$$
R_s = \frac{f_0}{8\pi CQ (f - f_0)^2}
$$
 (12)

The resistance is inversely proportional to the square of the frequency The characteristic impedance Z_0 has cancelled out. shift. **This equation cannot be solved exactly.** It has to be solved

The reactance far-off resonance is given approximately by

$$
X_s = \frac{I}{4\pi C (f_0 - f)}
$$
(13)

Because reactance far from resonance is independent of Q , it isn't necessary to have values of the Qs of the traps to make calculations involving resonance conditions.

A transmission line consisting of a wire of diameter d parallel to and at a distance H from a conducting plane has a characteristic impedance of

$$
Z_0 = 138 \text{ } LOG_{10} (4H/d) \text{ ohms} \tag{14}
$$

 H and d can be in any similar units.

A section of lossless transmission line of length S1 short-circuited at the far end has an input reactance of

$$
X_I = Z_0 \text{ TAN } A \tag{15}
$$

where

$$
A = \frac{360 \cdot S_I}{\lambda}
$$
 (16)
ssed as an angle in degrees.
on line of length S2 open-circuited at the
ctance of

$$
X_2 = \frac{-Z_0}{TAN B}
$$
 (17)

A is the length **S1** expressed as an angle in degrees.

A section of transmission line of length S2 open-circuited at the far end has an input reactance of

$$
X_2 = \frac{-Z_0}{TAN \, B} \tag{17}
$$

$$
\quad \text{where} \quad
$$

$$
B = \frac{360 \cdot S2}{\lambda} \tag{18}
$$

is an angle corresponding to the length S2.

Equation 1 gives the condition for resonance at a single frequency. It is necessary to derive from it an equation giving the condition for resonance at the two frequencies fl and f2. Note from eqns. 16 and 18 that angles A and B are inversely proportional to wavelength. Therefore they are, for fixed lengths S1 and S2, proportional to the frequency. Hence, if we now use A and *B* to denote the angles at the upper frequency f2, at fl they are reduced by the facreactances at these two frequencies are equal in magnitude but opposite in sign, it follows from eqn. 1 that:

tor f1/f2. Since the design is based on the assumption that the trap
reactances at these two frequencies are equal in magnitude but op-
posite in sign, it follows from eqn. 1 that:

$$
-TAN \frac{AfI}{f2} + \frac{I}{TAN} \frac{BfI}{f2} = (19)
$$

$$
TAN A - \frac{I}{TAN} \frac{BfI}{f2}
$$
The characteristic impedance Z_0 has cancelled out.

